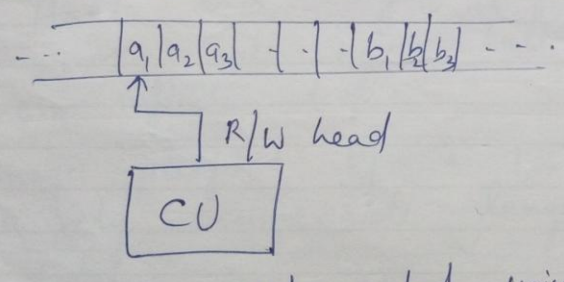
Turing Machine (TM) (Lecture -24) ①

- modified version of a PDA (more powerful)
- instead of a stack, it uses a tape to store symbols.
- TM is a generalized machine.
- can recognize all types of languages, viz RLs (RGs - type 3), CFL (CFGs or type 2), CSL (CSGs - type 1), type 0 or recursively enumerable languages

$\alpha \rightarrow \beta$, where $\alpha \in (V \cup T)^*$ (cannot be ε) and $\beta \in (V \cup T)^*$.

- type 0 grammar can only be recognized by a TM.

TM model



- TM can be represented using various notations such as
  - (i) TT (transition tables)
  - (ii) ID (Instantaneous descriptions)
  - (iii) TD (Transition diagrams).

Tape → divided into cells

- cell stores one symbol.

R/W head

- can read a symbol where it is pointing or write where it is pointing.

CU – reading/writing is determined by the CU.

- Different moves depend on current state and scanned symbol.
- TT.

Def TM $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

δ: $Q \times \Gamma$ to $Q \times \Gamma \times (L \times R)$

B: blank symbol.

Assumptions (TM we use)

- tape is divided into number of cells capable of storing 1 symbol
- M/c is deterministic (ie, it can have 0 or 1 transition for each configuration)
- Some of the symbols on the tape are input. When TM halts, symbols on tape are output.

Acceptance of a Language by TM

TM can do one of the following things:

(a) Halt and accept by entering into final state

(b) Halt and reject (if transition is not defined)

(c) TM will never halt and enters into an infinite loop

Language accepted by a TM is called recursively enumerable language or REL

Construction of a TM

a) $L = \{0^n 1^n \mid n \geq 1\}$ obtain TM.

Let w = 00001111

Sol Let q_0 be the start state and let the read-write head point to the first symbol of the string to be scanned.

Procedure

0000 1111

↓ ↓
X Y

- change state to q_1 and move right till we find a 1.
- when we find a 1, change state to q_2 and move ~~right~~ left till we find an X.

$\delta(q_0, 0) = (q_1, X, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_1, Y) = (q_1, Y, R)$

$\delta(q_1, 1) = (q_2, Y, L)$

$\delta(q_2, Y) = (q_2, Y, L)$

$\delta(q_2, 0) = (q_2, 0, L)$

$\delta(q_2, X) = (q_0, X, R)$.

XXXXYYYY

$\delta(q_0, Y) = (q_3, Y, R)$

$\delta(q_3, Y) = (q_3, Y, R)$

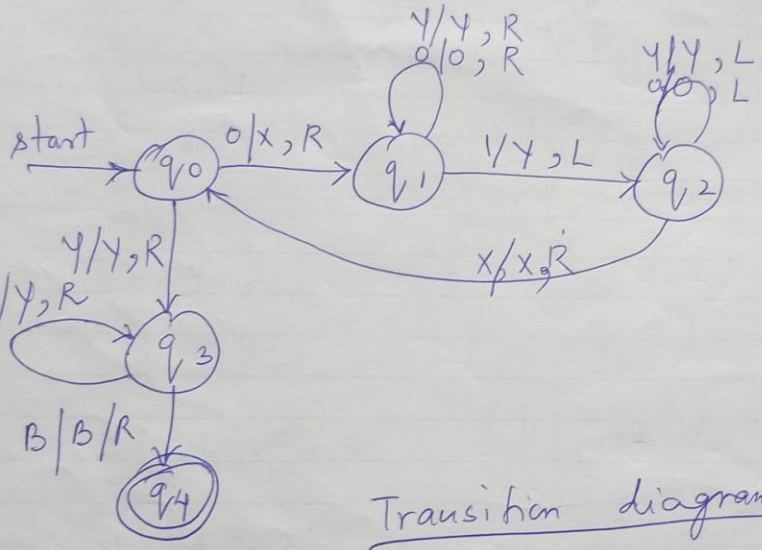
$\delta(q_3, B) = (q_4, B, R)$

States	Tape Symbols (Γ)				
δ	0	1	X	Y	B
q_0	(q_1, X, R)	–	–	(q_3, Y, R)	–
q_1	$(q_1, 0, R)$	(q_2, Y, L)	–	(q_1, Y, R)	–
q_2	$(q_2, 0, L)$	–	(q_0, X, R)	(q_2, Y, L)	–
q_3	–	–	–	(q_3, Y, R)	(q_4, B, R)
q_4	–	–	–	–	–

Transition diagram for TM

- TD consists of nodes corresponding to the states of a TM.
- An edge from state q to state p will have a label of the form $(X/Y, D)$, X and Y are tape symbols and D is the direction - L/R.

scanned symbol, written symbol.



Transition diagrams

accept string 0011

$q_0 0011 \vdash X q_1 011 \vdash X0 q_1 11 \vdash X q_2 0Y1 \vdash X q_0 0Y1$

$\vdash XX q_1 Y1 \vdash XXY q_1 1 \vdash XX q_2 YY$

$\vdash XX q_0 YY \vdash XXY q_3 Y \vdash XXYY q_3$

$\vdash XXYYB q_4$

↓ accepting / final state

4) Obtain a TM to accept the language (5)

$L(M) = \{ 0^n 1^n 2^n \mid n \geq 1 \}$.

Sol: n 0's followed by n 1's followed by n 2's.

q_0 000011112222BB

X Y Z

q_1 q_2 q_3 q_4

$\delta(q_0, 0) = (q_1, X, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_1, Y) = (q_1, Y, R)$

$\delta(q_1, 1) = (q_2, Y, R)$

$\delta(q_2, 1) = (q_2, 1, R)$

$\delta(q_2, Z) = (q_2, Z, R)$

$\delta(q_2, 2) = (q_3, Z, L)$

$\delta(q_3, Z) = (q_3, Z, L)$

$\delta(q_3, 1) = (q_3, 1, L)$

$\delta(q_3, Y) = (q_3, Y, L)$

$\delta(q_3, 0) = (q_3, 0, L)$

$\delta(q_3, X) = (q_0, X, R)$ XXOX4444ZZZ

q_0

$\delta(q_0, Y) = (q_4, Y, R)$

$\delta(q_4, Y) = (q_4, Y, R)$

$\delta(q_4, Z) = (q_5, Z, R)$

$\delta(q_5, Z) = (q_6, B, R)$

PS.

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$

$\Sigma = \{0, 1, 2\}$

$\Gamma = \{0, 1, 2, X, Y, Z, B\}$

q_0 - start state

B - blank character

~~F~~ $= q_6$.

δ is as in table:

state	Γ						
	0	1	2	Z	Y	X	B
q_0	q_1, X, R				q_4, Y, R		
q_1	$q_1, 0, R$	q_2, Y, R			q_1, Y, R		
q_2		$q_2, 1, R$	q_3, Z, L	q_2, Z, R			
q_3	$q_3, 0, L$	$q_3, 1, L$		q_3, Z, L	q_3, Y, L	q_0, X, R	
q_4				q_3, Z, R	q_4, Y, R		
q_5				q_3, Z, R			q_6, B, R
q_6